The Generalized Scattering Matrix Separation Technique Combined with the MM/FE Method for the Efficient Modal Analysis of a Comprehensive Class of 3D Passive Waveguide Circuits

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Abstract - **A novel and versatile technique for the efficient modal analysis of passive waveguide structures, called the generalized scattering matrix separation (GSMS) technique, is introduced. Its combination with the well-proven hybrid mode-matching/finite element (MM/FE) method achieves the fast and rigorous modal analysis of a comprehensive class of three-dimensional waveguide components which include homogeneous segments of arbitrarily shaped cross-section and one or more side coupled rectangular waveguide ports. This combined method takes the higher order mode coupling between all discontinuities or ports into account and requires only one frequency independent application of the two-dimensional finite element (FE) method. The method is verified by excellent agreement with measurements.**

I. Introduction

The availability of reliable and efficient CAD tools for waveguide components is of high importance for modern waveguide applications such as for space communication purposes [1]. In order to achieve the required accuracy, the consequent utilization of field-theory based models is indispensable, [2] - [12]. For the analysis of waveguide components of more general shape, pure numerical methods, such as the finite element method (FEM) [8], finite difference time domain (FDTD) [6] or frequency domain (FDFD) method [5] are typically used. Because of the required rather high numerical effort, however, these methods are not well appropriate for the CAD which calls very often for a rather large number of suitable optimization iteration steps. It is therefore highly desirable, to dispose of efficient but rigorous other methods which allow the reliable direct CAD of such waveguide structures on usual workstations.

For components composed of sections with homogenous rectangular and circular waveguide segments, the mode-matching method together with the generalized scattering or admittance matrix combination technique has already been applied for the successful CAD of many advanced structures [2], [9] -[12]. More recently, hybrid mode-matching finite difference [3] or finite element [4] methods have been presented as a flexible but efficient tool at the example of the CAD of waveguide two-ports including sections of more complicated structures. Multiport waveguide structures containing non-standard waveguide sections, such as ridged waveguides, may be analyzed by the three-plane mode-matching technique [7]. But this technique [7] has the disadvantage that the structure has to be analyzed three times and only the fundamental mode is considered in the port waveguides. There is a lack, therefore, in efficient but rigorous methods for the modal analysis and CAD of multi-port waveguide components containing arbitrarily shaped waveguide segments.

This paper presents a new and versatile technique, called the generalized scattering matrix separation (GSMS) technique. The combination of this technique with the mode-matching/finite element (MM/FE) method [4] achieves the fast and rigorous modal analysis and CAD of a comprehensive class of three-dimensional waveguide components which include homogeneous segments of arbitrarily shaped cross-section and one or more side coupled rectangular waveguide ports. To demonstrate the accuracy and efficiency of the presented method, structures of practical importance and considerable complexity are analyzed.

II. Theory

The principle of the generalized scattering matrix separation (GSMS) technique is based on the following idea: The modal generalized scattering matrix (GSM) (or generalized admittance matrix (GAM)) of the desired complicated component is considered to be composed of two simpler structures of which the modal GSM (or GAM) are either already known or can be computed more efficiently. The desired modal GSM (or GAM) is then calculated by subtracting the corresponding modal GSM (or GAM) of the simpler structures.

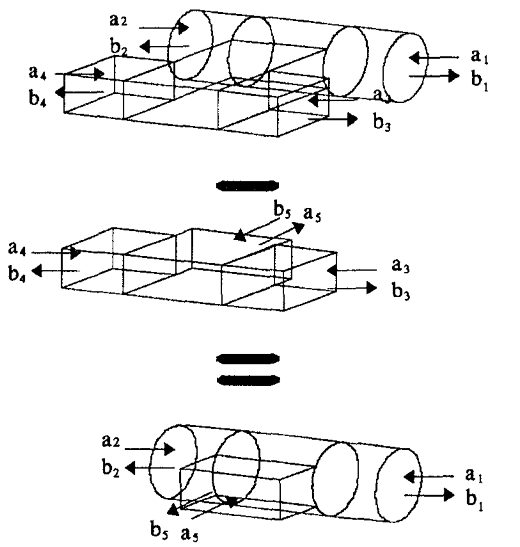

Fig. 1: Scattering matrix separation for a side coupled circular waveguide

The GSMS principle is illustrated in Fig. 1 for the example of a T-junction circular to side coupled rectangular waveguide. The analysis starts with the determination of the modal GSM of the computationally simpler four port device with two rectangular and two circular waveguide ports. This GSM can be computed in a straightforward manner by efficient methods, e.g. by the MM/FE method [4] since the four port structure contains merely single or multiple areas which are homogeneous in the same direction. The modal GSM of the rectangular waveguide T-junction (the second simpler structure) is already well-known [2]. The desired modal GSM of the circular waveguide T-junction with side coupled rectangular waveguide is then calculated by subtracting the corresponding modal GSMs of the simpler structures. The general formulas are given in equation 2.

For calculating the modal GSM of the simpler structures (e.g. the four port device in Fig.1), the hybrid mode-matching/finite element (MM/FE) method [4] is applied. The transverse electric $\vec{E}_t$ and magnetic fields $\vec{H}_t$ in each homogeneous waveguide section (e.g. of the four port device in figure 1) are represented by scalar potentials Ψ^H and Ψ^E [4] which are solutions of the transverse 2-dimensional homogeneous Helmholtz equation

$$\nabla_t^2 \, \Psi^{H,E} + k_c^2 \, \Psi^{H,E} = 0 \, , \qquad (1)$$

where k_c is the free space wave number. Ψ^H and Ψ^E have to satisfy homogeneous Dirichlet and Neumann boundary conditions on perfectly conducting electric Γ_E and magnetic walls Γ_M.

The Helmholtz equation is either solved analytically - for the circular and rectangular waveguide sections - or by application of the FE method [4] (e.g. in Fig.1 for the more general middle waveguide section shaped like a whistle). Matching the tranverse electromagnetic fields on the common interface at a waveguide step discontinuity (in this case a separate circular and rectangular area) leads to the general scattering matrix of the discontinuity [4], and the combination of the GSM's of subsequent discontinuities [9] yields the GSM of the complete four port device.

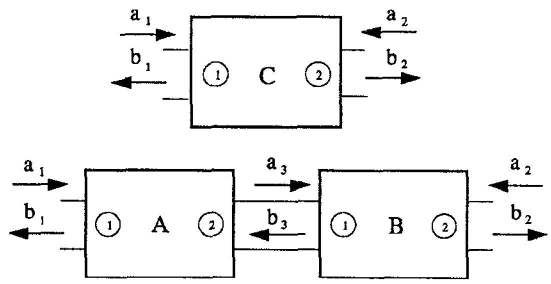

Fig. 2: Illustration of the scattering matrix separation technique

The GSMS technique is generally illustrated in Fig. 2. From the known scattering matrices $\mathbf{S}^C$ and $\mathbf{S}^B$, the elements of the scattering matrix $\mathbf{S}^A$ can be obtained by the following formulas:

$$\begin{aligned}
\mathbf{S}_{22}^A &= (\mathbf{S}_{11}^B - \mathbf{S}_{12}^B \, (\mathbf{S}_{22}^C - \mathbf{S}_{22}^B)^{-1} \, \mathbf{S}_{21}^B)^{-1} \, , \\
\mathbf{S}_{21}^A &= \mathbf{S}_{22}^A \, \mathbf{S}_{12}^B \, (\mathbf{S}_{22}^C - \mathbf{S}_{22}^B)^{-1} \, \mathbf{S}_{21}^C \, , \\
\mathbf{S}_{12}^A &= \mathbf{S}_{12}^C \, (\mathbf{S}_{22}^C - \mathbf{S}_{22}^B)^{-1} \, \mathbf{S}_{21}^B \, \mathbf{S}_{22}^A \, , \qquad (2) \\
\mathbf{S}_{11}^A &= \mathbf{S}_{11}^C + \mathbf{S}_{12}^C \, (\mathbf{S}_{22}^C - \mathbf{S}_{22}^B)^{-1} \, (\mathbf{S}_{21}^B \, \mathbf{S}_{21}^A - \mathbf{S}_{21}^C).
\end{aligned}$$

It should be noted that the matrices to be inverted in 2 become nearly singular if the number of modes at port 3 in Fig. 2 is chosen higher than the number of modes at port 2. On the other hand, for the analysis of usual structures, it has turned out to be sufficient to consider only all guided modes in structure B (e g. the H-plane T-junction, Fig. 1) together with some few lowest order evanescent modes

III. RESULTS

For the verification of the theory, several structures have been analyzed where either the hardware or measured data were already available. The first structure is the circular waveguide T-junction to side coupled rectangular $WR62$ waveguide. The scattering parameters of the structure are shown in Fig. 3.

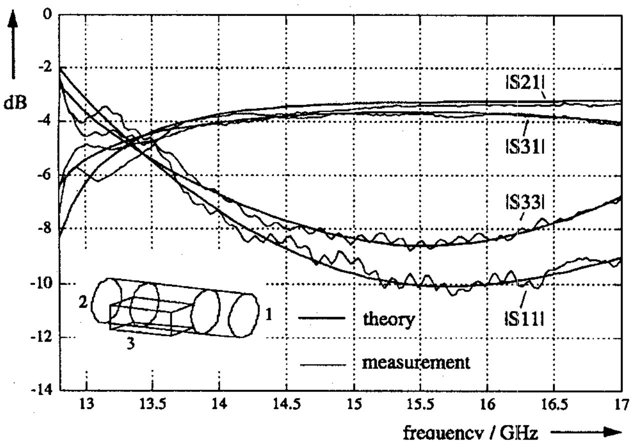

Fig. 3: Calculated and measured scattering parameters of a circular waveguide (radius $6.985mm$) with a side coupled $WR62$ waveguide ($15.799mm \times 7.899mm$).

Due to the symmetry of the structure, only the upper half was analyzed. The computation time for the analysis was approximately 12 sec. per frequency point on an IBM-RS6000-360 workstation. All modes up to 70 GHz cut-off frequency have been considered for the analysis of the 4-port device. The deviation of the scattering parameters at the beginning of the frequency range is due to the non-ideal taper used for the measurements which is used there close to its fundamental cut-off frequency.

For comparison, also the three-plane mode-matching technique of [7] has been used by us for analyzing the circular waveguide T-junction by applying the MM/FE method. The computation time was approximately three times higher and the the results are completely inaccurate in the near of and beyond the cut-off frequency of the TE_{20} rectangular waveguide mode.

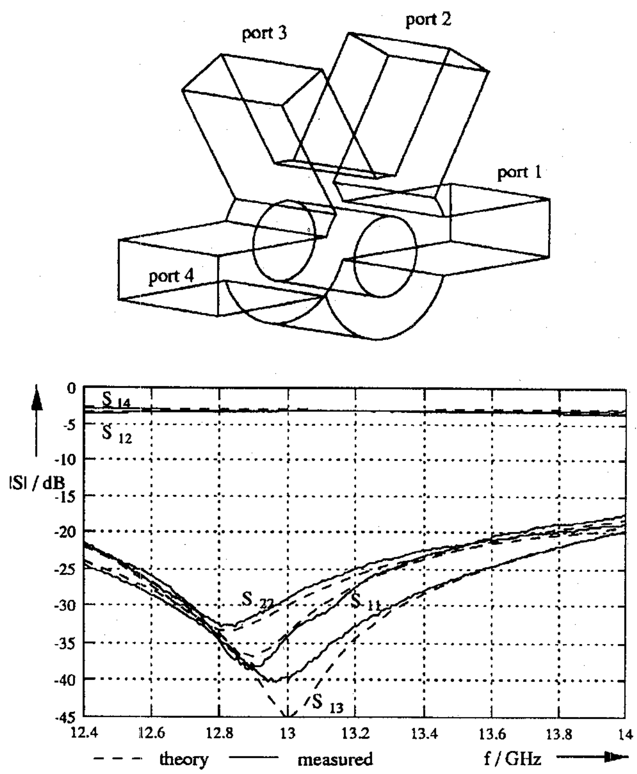

Fig. 4: Calculated and measured scattering parameters of a rat race structure

The second example is a rat race structure shown in Fig. 4 (rectangular waveguides $15.85 \times 7.9mm^2$ at $0°, 60°, 120°$, and $180°$, outer radius $10.875mm$ inner radius $5.35mm$). This structure was analyzed also by using T-junctions at each port. But in order to reduce the computation requirements, the even and odd midplane symmetry was utilized. The GSMS technique was applied to each port. Since the rat race is a planar circuit, there are also efficient other methods to analyse this structure, such as the boundary contour mode-matching method (BCMM) used in [10]. The GSMS technique, however, has the advantage, to be able to analyze also an insert with partial height only with one additional waveguide discontinuity. The computation time was approximately 20 sec. per frequency point. The BCMM-solution is not shown in Fig.4 because the difference to the solution by the proposed GSMS method is not distinguishable.

The third example shows a rectangular waveguide magic-T-junction (WR90 waveguides, $22.86 \times 10.16mm^2$). This structure was analyzed with a transition from 4 rectangular waveguides to a T-shaped waveguide back to 3 rectangular waveguides and application of the GSMS technique separation to ports 1, 2 and 3. The agreement between theory and measurements is almost as excellent as by pure modal analysis [11]. The consideration of all modes up to a cut-off frequency of 50 GHz was sufficient for convergence. The computation time was approxiomately 15 sec. per frequency point and is not increased if the cross section of port 4 is chosen arbitrarily or if the T-shaped waveguide is modified, e.g. to a Y-shaped waveguide.

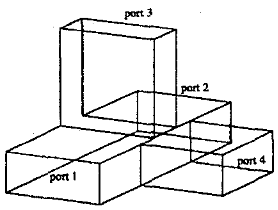

Fig. 5: Magic-T-junction

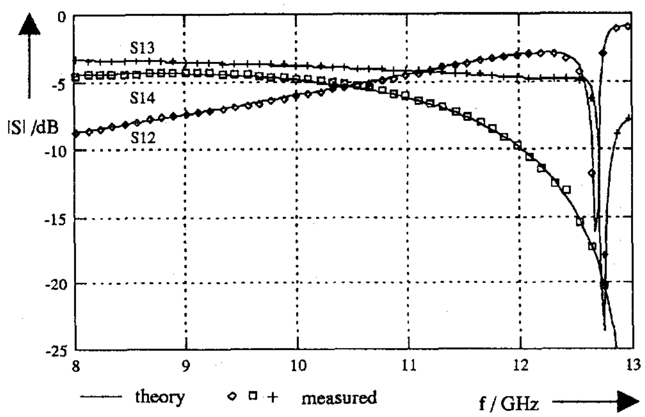

Fig. 6: Calculated and measured scattering parameters of the magic-T-junction

IV. Conclusion

This paper presents a novel and versatile technique for the efficient modal analysis of passive waveguide structures, called the generalized scattering matrix separation (GSMS) technique. This technique allows one to apply very efficiently appropriate field theory methods, such as the mode-matching or hybrid MM/FE method, to complicated microwave structures which could not be analyzed rigorously by fast techniques so far. It is shown that the combination of the GSMS technique with the well-proven hybrid mode-matching / finite element (MM/FE) method achieves the fast and rigorous modal analysis of a comprehensive class of three-dimensional waveguide components which include homogeneous segments of arbitrarily shaped cross-section and one or more side coupled rectangular waveguide ports. The method is verified by excellent agreement with measurements.

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